# Partitioning Edge-Coloured Graphs into Monochromatic Paths and Cycles

Jess Ryan

# Definitions

## Edge-colouring

A graph $G$ is $r$-edge-coloured if its edges are coloured using at most $r$ colours.

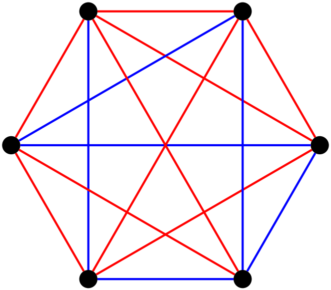

Figure: **2-edge-coloured** $K_6$

# Where it all started...

**Question:**

Can we partition the vertices of an edge-coloured graph into a number of monochromatic subgraphs depending only on the number of colours used (and not on $n$)?

**Conjecture (Gyárfás, 1989)**

At most $r$ disjoint monochromatic paths are needed to partition the vertices of any $r$-edge-coloured $K_n$.

**Conjecture (Erdős, Gyárfás and Pyber, 1991)**

At most $r$ disjoint monochromatic cycles are needed to partition the vertices of any $r$-edge-coloured $K_n$.

# Partitioning $K_n$ into paths

**Conjecture (Gyárfás, 1989)**

At most $r$ disjoint monochromatic paths are needed to partition the vertices of any $r$-edge-coloured $K_n$.

- $r = 1$ ✓ (trivial)
- $r = 2$ ✓ (Gyárfás and Gerencsér, 1967)

# $r = 2$ proof:

Basic idea:

- Construct a maximal red path followed by a maximal blue path.
- While there are uncovered vertices, extend the path while maintaining the single colour change property.
- Continue until all vertices are covered.

# Partitioning $K_n$ into paths

**Conjecture (Gyárfás, 1989)**

At most $r$ disjoint monochromatic paths are needed to partition the vertices of any $r$-edge-coloured $K_n$.

- $r = 1$ ✓ (trivial)
- $r = 2$ ✓ (Gyárfás and Gerencsér, 1967)
- $r = 3$ ✓ (Pokrovskiy, 2014)
- $r \geq 4$ ?

# Partitioning $K_n$ into cycles

**Conjecture (Erdös, Gyárfás and Pyber, 1991)**

At most $r$ disjoint monochromatic cycles are needed to partition the vertices of any $r$-edge-coloured $K_n$.

- $r = 1$ ✓ (trivial)
- $r = 2$ ✓ (Bessy and Thomassé, 2010)
- $r \geq 3$ ✗ (Pokrovskiy, 2014)

**Conjecture (Pokrovskiy, 2014)**

The vertices of any $r$-edge-coloured $K_n$ can be partitioned into $r$ (not necessarily disjoint) monochromatic cycles.

**Conjecture (Pokrovskiy, 2014)**

At most $r$ disjoint monochromatic cycles are needed to cover all but $c_r$ of the vertices of any $r$-edge-coloured $K_n$.

## Local-colouring

An edge colouring of a graph $G$ is an $r$-local-colouring if each vertex in $G$ is incident to at most $r$ distinct colours.

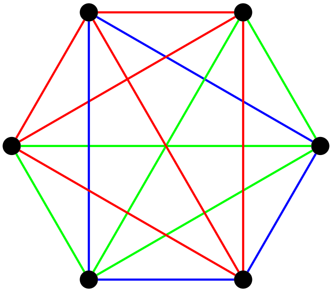

Figure: **2-local colouring of $K_6$**

# Definitions

## Mean-colouring

An edge colouring of a graph $G$ is an $r$-mean colouring if the mean number of colours incident to a vertex in $G$ is equal to $r$.

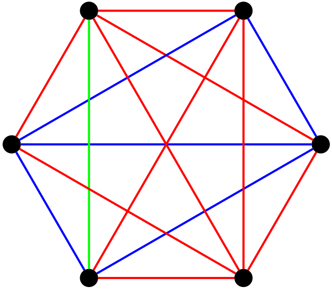

Figure: **2-mean colouring of $K_6$**

# Partitioning $K_n$ into cycles

**Theorem (Bessy & Thomassé, 2010)**

The vertices of any 2-edge-coloured $K_n$ can be partitioned into 2 disjoint cycles of different colours.

**Theorem (Conlon & Stein, 2014)**

The vertices of any 2-locally coloured $K_n$ can be partitioned into 2 disjoint cycles of different colours.

**Theorem (Conlon & Stein, 2014)**

The vertices of any 2-mean coloured $K_n$ can be partitioned into 2 disjoint cycles of different colours.

# Partitioning $K_{n,n}$ into paths

**Conjecture (Pokrovskiy, 2014)**

At most $2r - 1$ disjoint monochromatic paths are needed to partition the vertices of any $r$-edge-coloured $K_{n,n}$.

- $r = 2$ ✓ (Pokrovskiy, 2014)

**Theorem (Lang & Stein, 2015)**

The vertices of any 2-locally coloured $K_{n,n}$ can be partitioned into 3 disjoint monochromatic paths.

**Conjecture**

The vertices of any 2-mean coloured $K_{n,n}$ can be partitioned into 3 disjoint monochromatic paths.

# Partitioning $K_{n,n}$ into cycles

## Conjecture

The vertices of any 2-edge-coloured $K_{n,n}$ can be partitioned into 3 disjoint monochromatic cycles.

## Conjecture

The vertices of any 2-locally coloured $K_{n,n}$ can be partitioned into 3 disjoint monochromatic cycles.

## Conjecture

The vertices of any 2-mean coloured $K_{n,n}$ can be partitioned into 3 disjoint monochromatic cycles.